SNR IMPROVEMENT IN DPCM OVER PCM:

DPCM System:

Where,
$m[k] = k^{th}$ sample of the samples analog signal, $m_q[k] = k^{th}$ quantized sample

$
\hat{m}_q[k] = \text{predicted value of } m_q[k]
$

$d[k] = \text{Prediction Error} = m[k] - \hat{m}_q[k],$

$d_q[k] = \text{Quantized prediction error} = d[k] + q[k] = \text{Prediction Error} + \text{Quantization Error}$
Let \( m_p \) = peak amplitude of \( m(t) \) in the case of PCM 
& \( d_p \) = peak amplitude of \( d(t) \) in case of DPCM

\( L \) = # of levels used in Quantizer for both the cases

Then, 
\[
\Delta_{\text{PCM}} = \Delta_v = \text{Step for PCM} = \frac{2m_p}{L}
\]
& 
\[
\Delta_{\text{DPCM}} = \Delta_v = \text{Step for DPCM} = \frac{2d_p}{L}
\]

Therefore, \( \Delta_{v2} = \frac{d_p}{m_p} \Delta_{v1} \)

**It means the step for DPCM is reduced by a factor of \( \frac{d_p}{m_p} \).**

The Quantization Noise Power for PCM \( = \sigma_Q^2 \) = \( \frac{\Delta_v^2}{12} = \frac{m_p^2}{3L} \)

The Quantization Noise Power for DPCM \( = \sigma_{Q2}^2 \) = \( \frac{\Delta_v^2}{12} = \frac{d_p^2}{3L} \)

That means the Quantization noise power in DPCM is reduced by a factor of \( \frac{m_p^2}{d_p^2} \) and SNR increases by the same factor.

The signal-to-noise ratio for DPCM is given by:

\[
(SNR)_Q = \frac{\sigma_M^2}{\sigma_Q^2}
\]

where \( \sigma_M^2 \) = variance of the original signal \( m(t) \)

and \( \sigma_Q^2 \) = variance of the quantization error \( q[k] \)

The above equation can be rewritten as:

\[
(SNR)_Q = \frac{\sigma_M^2}{\sigma_Q^2} = \left( \frac{\sigma_M^2}{\sigma_d^2} \right) \left( \frac{\sigma_d^2}{\sigma_Q^2} \right).
\]

Where, \( \sigma_d^2 \) = variance of the prediction error \( d[k] \).

\( (SNR)_Q \) = Signal-to-Quantization noise ratio for DPCM because the input signal to the quantizer is the prediction error \( d[k] \).

and \( G_p \) = Processing gain = SNR improvement due to prediction = \( \frac{\sigma_M^2}{\sigma_d^2} \)

Now for a given base-band (message) signal \( \sigma_M^2 \) is fixed, so that \( G_p \) is maximized by minimizing the variance \( \sigma_d^2 \) of the prediction error \( d[k] \). So our goal should be to design a prediction filter so as to minimize \( \sigma_d^2 \) and that can be achieved by increasing the order of the prediction filter. But for voice signals it is found that the greatest improvement occurs in going from no prediction to first-order prediction, with some additional gain resulting from increasing the order of the prediction filter up to 4 or 5, after which little additional gain is obtained.

**e.g** If the signal-to-quantization noise ratio for a standard PCM system and a DPCM system is 30 db. The value of \( \alpha \) for DPCM is 10. A signal is sampled at a rate of 8kHz. Determine how much of bps saving is achieved by using DPCM instead of standard PCM.

\[
(SNR)_Q = \alpha + 6 = 1.8 + 6n_1 = 30 \text{ dB for PCM}
\]

\[
(SNR)_Q = \alpha + n_2 = 10 + 6n_2 = 30 \text{ dB for DPCM}
\]

\[\Rightarrow 6n_1 = 30 - 1.8 = 28.2 \text{ dB} \]

\[\Rightarrow 6n_2 = 30 - 10 = 20 \text{ dB} \]

\[\Rightarrow (n_1-n_2) = 8.2/6 = 1.3 \approx 2 \text{ bits/sample saved using DPCM} \]

Saving in bps = \( (n_1-n_2) f_s \) = 2 \times 8 kHz = \text{16 kb/s}
DELTA MODULATION (DM):
The correlation between signal samples used in DPCM can be further exploited by oversampling (typically 4 times the Nyquist rate) in Delta Modulation (DM). This increases the correlation between adjacent samples, which results in a small prediction error using only one bit (L=2). DM is basically a 1-bit DPCM Delta modulation is a special case of DPCM in that it uses first order prediction filter as follows:

DM System:

\[ m[k] \]

\[ \sum \]

\[ d[R] \]

Quantizer

\[ d_q[k] \]

To channel

\[ m_q[k-1] \]

Delay \( T_s \)

\[ m_q[k] \]

**Transmitter (Modulator)**

\[ m_q[k] = m_q[k-1] + d_q[k] \]

\[ m_q[k-1] = m_q[k-2] + d_q[k-1] \]

Therefore,

\[ m_q[k] = m_q[k-1] + d_q[k] = m_q[k-2] + d_q[k-1] + d_q[k] \]

If we go on doing this iteratively and assume initial conditions to be zero, i.e. \( m[0]=0 \),

Then

\[ m_q[k] = \sum_{i=0}^{k} d_q[i] \]
**DELTA MODULATION:**

- **PCM:** Analog signal samples are quantized in $L$ levels, and this information is transmitted by $n$ pulses per sample ($n = \log_2 L$).
- **DM:** The modulated signal carries information about the difference between successive samples. Basically, therefore, DM carries the information about the derivative of $m(t)$, and hence it is called *Delta Modulation.*

$$d_q[k] = A \text{ delta modulated signal, its integration yields } m_q(t), \text{ which is approximation of } m(t).$$
Problems of DM:

1. Coding Threshold:
Variations in $m(t)$ smaller than the step size, $\sigma$, are lost in DM. This is called Threshold of Coding.

2. Slope-Overload:
If $m(t)$ changes too fast, i.e., $m(t)$ is too high, $m_q(t)$ cannot follow $m(t)$, and overloading occurs. This is called Slope-Overload and it gives rise to slope-overload noise in DM. The granular nature of the output signal produces granular noise similar to the quantization noise. The slope-overload noise can be reduced by increasing $\sigma$ (the step size). But this increases the granular noise.

There is an optimum value of $\sigma$, which yields the best compromise giving the minimum overall noise.

*The optimum value of $\sigma$ depends on the sampling frequency, $f_s$ and the nature of the signal*

$m_q(t) = \text{Integration of } d_q[k] \text{ cannot follow } m(t)$.

During the sampling interval $T_s$, $m_q(t)$ is capable of changing by $\sigma$

Where, $\sigma = \text{Step size = Height of step}$

Therefore, maximum slope $m_q(t)$ can follow $\frac{\sigma}{T_s} = \sigma f_s$

Hence no overload occurs if

$$\left| m(t) \right| \leq \sigma f_s$$

e.g., $m(t) = A \cos \omega t \rightarrow \left| m(t) \right|_{\text{max}} = \omega A \leq \sigma f_s$ for no overload condition.

Hence, the maximum amplitude $A_{\text{max}}$ of this signal that can be tolerated without overload is given by:

$$A_{\text{max}} \equiv \sigma f_s / \omega$$

$$[A_{\text{max}}]_{\text{voice}} \equiv \sigma f_s / \omega_r, \text{ where } \omega_r = 2\pi \times 800$$